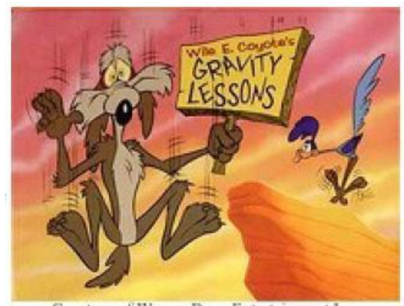
**Projectiles Launched Horizontally**

The study of projectile motion brings together a lot of what you have learned in the past few lessons. You need to know about gravity, velocity, acceleration, and vector components to be able to fully understand and solve the upcoming questions. Don't worry, though. Even with all this material to keep track of, learning how to do these questions a little bit at a time makes it all manageable.

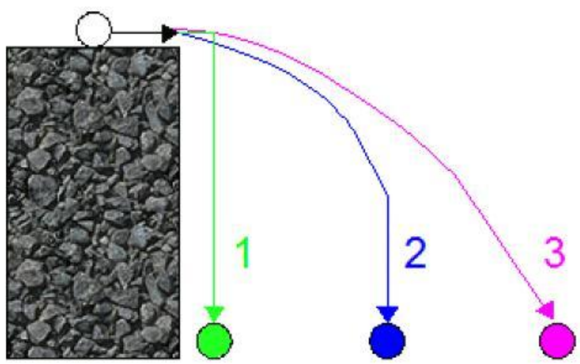
**The Wile E. Coyote Effect**

I'm sure you have seen the cartoon where the Coyote is chasing after the Road Runner and runs off a cliff. He hangs in mid air for a second, looks down, and then starts to fall. The question is how true is this and how many people believe it to be true?




Courtesy of Warner Bros. Entertainment Inc.

Question:
"Ignoring air resistance, which of the following correctly shows what an object would do if it rolled off a cliff?"




1. The object will stop in midair and then start to fall straight down.
2. The object will move forward at first but will eventually just fall straight down.
3. The object will continue to move forwards the entire time it is falling.

The correct answer is number 3, which should make sense since there is nothing pushing against the ball to slow it down in the horizontal direction.

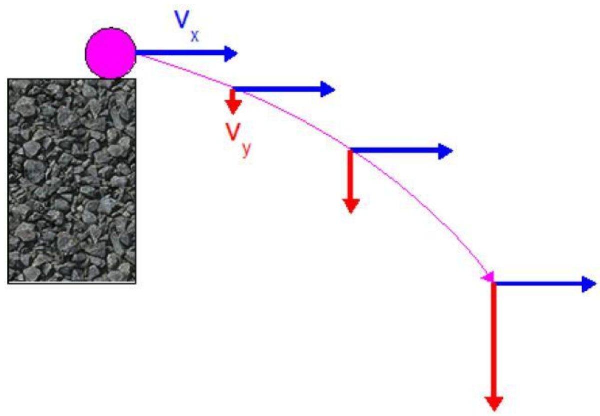
● If we are ignoring air resistance then there is no horizontal force to cause a horizontal acceleration.
● Since there is no horizontal acceleration, the coyote (or the ball) will travel horizontally at the same speed the whole time!

This, however, does not tell us anything at all about what is happening vertically, which is completely separate from what the object is doing horizontally.

● As soon as the coyote leaves the cliff he will experience a vertical force due to gravity.
● This force will cause him to start accelerating in the vertical direction.
● As he falls he will be going faster and faster in the vertical direction.

If you look at this problem as two separate problems (i.e. look at just what's happening horizontally, and then look at what's just happening vertically), you should get the idea that this is just like the component vectors you were working on earlier in the course.

● The horizontal and vertical components of the motion of an object going off a cliff are separate from each other, and do not affect each other.
● In virtually all textbooks you will see the horizontal component called "x" and the vertical component called "y". If we look at the horizontal and vertical components of an object's velocity as it rolls off a cliff, we would get something that looks like this:




● The x-component of the velocity is there from the start, and stays exactly the same the entire time.
● The y-component doesn't even exist at the beginning, but grows bigger and bigger as the object falls.
● The shape of the path that the ball follows is a **parabola**. If you've studied these in math class, great! But don't worry about it if you haven't, just as long as you recognize the shape and know the name.

To understand how to actually figure out questions involving these situations, it's probably best to look at an example.

● When you are doing a part of a question that has to do with vertical movement "think vertical" and only use vertical ideas (like gravity).
● When you are doing a part of a question that has to do with horizontal movement "think horizontal" and only use horizontal ideas (no gravity / no acceleration).

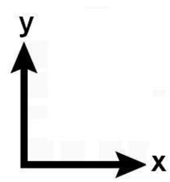
**Example 1**

A ball is thrown off the edge of a 15.0m tall cliff horizontally at 8.0m/s.
a) Determine how much time it takes to fall.
b) Determine how far from the base of the cliff the ball hits the ground.
c) Determine how fast it is moving vertically when it hits the ground.
d) Determine what its total velocity is when it hits the ground.

**a) "Think vertical" – Determine how much time it takes to fall.**
Since we are talking about something falling, and this is vertical motion, we will only use vertical ideas.

The easiest way to solve this question is to make use of the fact that it takes the exact same amount of time for the ball to hit the ground if we were to drop it straight off the cliff as it would if we rolled it off the cliff. With this in mind, let's calculate the time it would take for the ball to fall 15m.

Begin by choosing a coordinate system. I'll choose up and right to be positive, and down and left to be negative as shown below:




Next, write down the information:

$v_{iy}$ = 0m/s (Note that this refers to the initial velocity in the y direction)
$d_y$ = -15.0m (Note that "$d_y$" represents the ball's vertical displacement, and since the ball is falling downwards it will be negative)
$a_y$ = -9.80m/s$^2$ (Since the ball is accelerating downwards it will be negative)
t = ?

$$d_y = v_{iy}t + \tfrac{1}{2}a_y t^2 = (0m/s)t + \tfrac{1}{2}a_y t^2$$

$$d_y = \tfrac{1}{2}a_y t^2$$

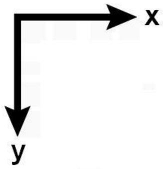
$$\frac{2d_y}{a_y} = t^2$$

$$\sqrt{\frac{2d_y}{a_y}} = \sqrt{t^2}$$

$$\sqrt{\frac{2d_y}{a_y}} = t$$

$$t = \sqrt{\frac{2d_y}{a_y}} = \sqrt{\frac{2(-15.0m)}{-9.80m/s^2}} = 1.75s$$

As shown above, we found the time it takes for the ball to hit the ground to be 1.75s. However, you might be curious to know what would have happened if we had chosen down and right to be positive, as shown here:



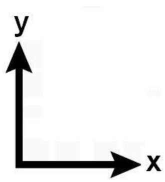

This would have made it so that $d_y$ and $a_y$ were both positive quantities, so the final line of the question would have looked as follows:

$$t = \sqrt{\frac{2d_y}{a_y}} = \sqrt{\frac{2(15.0m)}{9.80m/s^2}} = 1.75s$$

Notice that we get the same answer regardless of the coordinate system we choose.

**b) "Think horizontal" – Determine how far from the base of the cliff the ball hits the ground.**

Note for the remainder of this question I will use right and up to be my positive x and y directions, but feel free to use right and down if you prefer doing your questions this way.




We know that the ball was in the air for 1.75s (from the previous question), and it was moving at a constant speed of 8.0m/s in the x-direction the whole time, so…

Begin by writing down the information:

t = 1.75s
$d_x$ = ?
$v_{ix} = v_{fx} = v_x$ = 8.0m/s Note that since the initial and final velocities in the horizontal direction are the same, we could use any of the three symbols $v_{ix}$, $v_{fx}$, or $v_x$ to represent the velocity in the x-direction. Note that with velocities in the y direction this does not work since there is an acceleration (gravity)!

$$v_x = \frac{d_x}{t}$$
$$v_x t = d_x$$
$$d_x = v_x t = (8.0m/s)(1.75s) = 14m$$

**c) "Think vertical" – Determine how fast it is moving vertically when it hits the ground.**

Begin by writing down the information:

$v_{iy}$ = 0m/s
$v_{fy}$ = ?
$d_y$ = -15.0m
$a_y$ = -9.80m/s$^2$ (Since the ball is accelerating downwards it will be negative)

$$v_{fy}{}^2 = v_{iy}{}^2 + 2a_y d_y$$
$$v_{fy} = \sqrt{v_{iy}{}^2 + 2a_y d_y}$$
$$v_{fy} = \sqrt{(0m/s)^2 + 2(-9.80m/s^2)(-15.0m)} = 17.1m/s$$

However, since we picked up to be positive, and down to be negative, $v_{fy}$ = -17.1m/s. Remember that when you take a square root, there is always a positive and a negative root as shown below.

$$v_{fy} = \pm\sqrt{(0m/s)^2 + 2(-9.80m/s^2)(-15.0m)} = \pm 17.1m/s$$
$$v_{fy} = -17.1m/s$$

We must pick the negative root for our answer to make sense

**d) "Combine horizontal and vertical" – Determine what its total velocity is when it hits the ground.**

Its total velocity is found by adding the final horizontal and vertical components of the velocity together.

As always, begin by writing down the information:

$v_{fx}$ = 8.0m/s
$v_{fy}$ = -17.1m/s

$$a^2 + b^2 = c^2$$

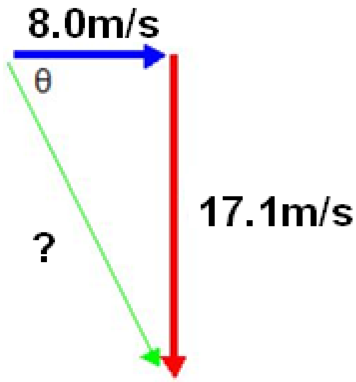
$$\sqrt{a^2 + b^2} = \sqrt{c^2}$$

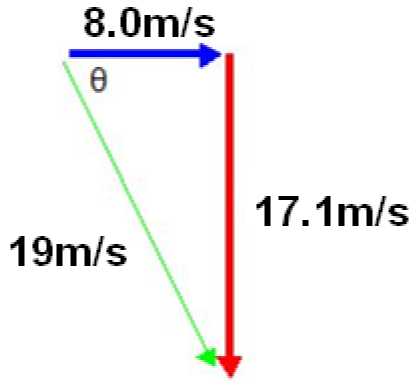
$$\sqrt{a^2 + b^2} = c$$

$$\sqrt{{v_{fx}}^2 + {v_{fy}}^2} = v_{total}$$

$$v_{total} = \sqrt{{v_{fx}}^2 + {v_{fy}}^2} =$$

$$v_{total} = \sqrt{(8.0m/s)^2 + (-17.1m/s)^2} = 19m/s$$



If we wanted to find the angle, theta, we would use an inverse trig function. It doesn't matter which inverse trig function we use, the value of theta should be the same. As you can see below, the only reason we aren't getting the same answer is because our answers are rounded.

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{adj}{hyp} = \frac{8.0m/s}{19m/s}$$

$$\cos\theta = \frac{8.0m/s}{19m/s}$$

$$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{8.0m/s}{19m/s}\right)$$

$$\theta = \cos^{-1}\left(\frac{8.0m/s}{19m/s}\right) = 65°$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{17.1m/s}{8.0m/s}$$

$$\tan\theta = \frac{17.1m/s}{8.0m/s}$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{17.1m/s}{8.0m/s}\right)$$

$$\theta = \tan^{-1}\left(\frac{17.1m/s}{8.0m/s}\right) = 65°$$

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{opp}{hyp} = \frac{17.1m/s}{19m/s}$$

$$\sin\theta = \frac{17.1m/s}{19m/s}$$

$$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{17.1m/s}{19m/s}\right)$$

$$\theta = \sin^{-1}\left(\frac{17.1m/s}{19m/s}\right) = 64°$$

To summarize there are two ways to answer this question:

1. $v_x$ = 8.0m/s, $v_y$ = -17.1m/s.
2. $v_{total}$ = 19m/s at 65° below the horizontal (or 64° if you used sine).