

## Lesson 6 Worksheet

1. A base ball is thrown with an initial speed of 20 m/s at an angle of  $10^\circ$  above the horizontal.


Determine:

- How long it is in the air for. ( $t = 0.71s$ )
- What its maximum height will be. ( $0.61\text{ m}$ )
- How long the range (horizontal displacement) is. ( $14\text{ m}$ )

<p>a.</p> <p><math>v_i = 20\text{m/s}</math>  <math>v_x = 20\text{m/s}\cos 10^\circ = 19.7\text{m/s}</math>  <math>v_{iy} = (20\text{m/s})\sin 10^\circ = 3.47\text{m/s}</math>  <math>v_{fy} = -3.47\text{m/s}</math>  <math>a_y = -9.80\text{m/s}^2</math>  <math>t = ?</math></p>	$a_y = \frac{v_{fy} - v_{iy}}{t}$ $t = \frac{v_{fy} - v_{iy}}{a_y}$ $t = \frac{(-3.47\text{m/s}) - (3.47\text{m/s})}{(-9.80\text{m/s}^2)} = 0.71\text{s}$
<p>b.</p> <p><math>v_{iy} = 3.47\text{m/s}</math>  <math>v_{fy} = 0\text{m/s}</math>  <math>a_y = -9.80\text{m/s}^2</math>  <math>d_y = ?</math></p>	$v_f^2 = v_i^2 + 2ad$ $v_f^2 - v_i^2 = 2ad$ $\frac{v_f^2 - v_i^2}{2a} = d$ $d = \frac{v_f^2 - v_i^2}{2a} = \frac{(0\text{m/s})^2 - (3.47\text{m/s})^2}{2(-9.80\text{m/s}^2)}$ $d_y = 0.61\text{m}$
<p>c.</p> <p><math>v_{ix} = v_{fx} = v_x = 19.7\text{m/s}</math>  <math>t = 0.71\text{s}</math>  <math>d_x = ?</math></p>	$d_x = \left( \frac{v_{fx} + v_{ix}}{2} \right) t = \left( \frac{19.7\text{m/s} + 19.7\text{m/s}}{2} \right) (0.71\text{s})$ $d_x = 14\text{m}$

2. An artillery shell is launched at an unknown angle. It strikes the ground (at the height that it was fired) 250 m away and has a maximum height of 100 m. Determine:

- The initial vertical velocity. (44.3 m/s)
- The time the shell is in the air. (9.04 s)
- The horizontal velocity. (27.7 m/s)
- The initial speed. (52.2 m/s)
- The angle the projectile was launched at. (58.0°)
- What is the velocity 2.00 seconds after launch?
- What is the velocity on impact?

<p>a. Let's start by writing down all the information we can about the half way point of the projectile's flight.</p> <p><math>d_{y\max} = 100\text{m}</math>  <math>a_y = -9.80\text{m/s}^2</math>  <math>v_{iy} = ?</math>  <math>v_{fy} = 0\text{m/s}</math></p>	$v_{fy}^2 = v_{iy}^2 + 2a_y d_{y\max}$ $v_{fy}^2 - 2a_y d_{y\max} = v_{iy}^2$ $v_{iy}^2 = v_{fy}^2 - 2a_y d_{y\max}$ $\sqrt{v_{iy}^2} = v_{iy} = \sqrt{v_{fy}^2 - 2a_y d_{y\max}}$ $v_{iy} = \sqrt{(0\text{m/s})^2 - 2(-9.80\text{m/s}^2)(100\text{m})}$ $v_{iy} = 44.3\text{m/s}$
<p>b.</p> <p><math>v_{iy} = 44.3\text{m/s}</math>  <math>v_{fy} = -44.3\text{m/s}</math>  <math>t = ?</math></p>	$a_y = \frac{v_{fy} - v_{iy}}{t}$ $t = \frac{v_{fy} - v_{iy}}{a_y} = \frac{-44.3\text{m/s} - 44.3\text{m/s}}{-9.80\text{m/s}^2} = 9.04\text{s}$
<p>c.</p> <p><math>t = 9.04\text{s}</math>  <math>d_x = 250\text{m}</math>  <math>v_{ix} = v_{fx} = v_x = ?</math></p>	$v_x = \frac{d_x}{t} = \frac{250\text{m}}{9.04\text{s}} = 27.7\text{m/s}$
<p>d.</p> <p><math>v_{ix} = 27.7\text{m/s}</math>  <math>v_{iy} = 44.3\text{m/s}</math></p>	$v_i^2 = v_{ix}^2 + v_{iy}^2$ $v_i = \sqrt{v_{ix}^2 + v_{iy}^2}$ $v_i = \sqrt{(27.7\text{m/s})^2 + (44.3\text{m/s})^2} = 52.2\text{m/s}$
<p>e. Draw a triangle with the vectors <math>v_{ix}</math>, <math>v_{iy}</math>, and <math>v_i</math> and then use sin, cos, or tan to solve. I'll use tan.</p>  <p>*** For some reason the computer messed up the inverse tan part. All I did was take the inverse tan of both sides, so inverse tan of 1.60 = 58.0°</p>	$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{v_{iy}}{v_{ix}} = \frac{44.3\text{m/s}}{27.7\text{m/s}}$ $\tan \theta = 1.60$ $\tan^{-1}(\tan \theta) = \tan^{-1}(1.60)$ $\theta = 58.0^\circ$

<p>f.</p> <p><math>v_{iy} = 44.3\text{m/s}</math>  <math>v_{fy} = ?</math>  <math>a_y = -9.80\text{m/s}^2</math>  <math>t = 2.00\text{s}</math></p> <p><math>v_x = 27.7\text{m/s}</math> (it never changes)</p> <p>So the velocity when <math>t = 2.00\text{s}</math> is:  <math>v_x = 27.7\text{m/s}</math>  <math>v_y = 24.7\text{m/s}</math></p>	$a_y = \frac{v_{fy} - v_{iy}}{t}$ $a_y t = v_{fy} - v_{iy}$ $a_y t + v_{iy} = v_{fy}$ $v_{fy} = a_y t + v_{iy}$ $v_{fy} = (-9.80\text{m/s}^2)(2.00\text{s}) + 44.3\text{m/s} = 24.7\text{m/s}$
<p>g. We could do a whole bunch of calculations, but why bother? The easiest way to answer this question is to understand that <math>v_x</math> will not change, and <math>v_{fy}</math> is the opposite of <math>v_{iy}</math>.</p> <p><math>v_{fx} = 27.7\text{m/s}</math>  <math>v_{fy} = -44.3\text{m/s}</math></p>	