Lesson 5 Worksheet

1. A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000 m with a forward velocity of 100 m/s. If air friction is ignored, how far in advance of the starving explorer's drop zone should she release the package?

This question is asking us to determine the range of the projectile, d_x. To find the horizontal distance we'll need to find out how long the package is in flight. d_y = -2000m v_{iy} = 0m/s a_y = -9.80m/s^2 t = ?	$d = v_i t + \frac{1}{2}at^2$ $d = 0 + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$ $\frac{2d}{a} = t^2$ $t^2 = \frac{2d}{a}$ $\sqrt{t^2} = t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-2000m)}{(-9.80m/s^2)}} = 20.2s$
Now we can find the horizontal distance. v_x = 100m/s t = 20.2s	$v_x = \frac{d_x}{t}$ $v_x t = d_x$ $d_x = v_x t = (100m/s)(20.2s) = 2.02 \times 10^3 m$

2. A rifle is fired horizontally from 1.90 m above the ground. The bullet is found to have travelled 200 m. Ignoring air friction, at what speed must the bullet have been travelling as it left the barrel?

This question is asking us to determine the initial speed of the bullet. Since the gun was fired horizontally, this is the same as asking us to find v_x. d_y = -1.90m v_{iy} = 0m/s a_y = -9.80m/s^2 t = ?	$d = v_i t + \frac{1}{2}at^2$ $d = 0 + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$ $\frac{2d}{a} = t^2$ $t^2 = \frac{2d}{a}$ $\sqrt{t^2} = t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-1.90m)}{(-9.80m/s^2)}} = 0.623s$
Now we can find the horizontal velocity. d_x = 200m t = 0.388s	$v_x = \frac{d_x}{t} = \frac{200m}{0.623s} = 321m/s$

3. A ski jumper leaves the horizontal end of the ramp with a velocity of 25 m/s and lands 70 m from the base of the ramp. How high is the end of the ramp above the landing area?

We're trying to find d_y. d_x = 70m v_x = 25m/s ax t = ?	$v_x = \frac{d_x}{t}$ $t = \frac{d_x}{v_x} = \frac{70m}{25m/s} = 2.8s$
Now we can find the horizontal velocity. v_{iy} = 0m/s a_y = -9.80m/s^2 t = 2.8s d_y = ?	$d = v_i t + \frac{1}{2}at^2$ $d = 0 + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$ $d = \frac{1}{2}(-9.80m/s^2)(2.8s)^2$ $d = -38m$ $d_y = -38m$

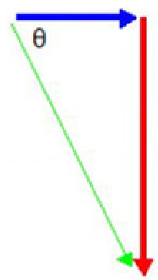
4. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.
a. How long does it take the stone to reach the bottom of the cliff?
b. How far from the base of the cliff does the stone hit the ground?
c. What are the horizontal and vertical components of the stone's velocity just before it hits the ground?

a. d_y = -78.4m v_{iy} = 0m/s a_y = -9.80m/s^2 t = ? Note, you could also let d_y and a_y be positive if you define down to be the positive y direction.	$d = v_i t + \frac{1}{2}at^2$ $d = 0 + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$ $\frac{2d}{a} = t^2$ $t^2 = \frac{2d}{a}$ $\sqrt{t^2} = t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-78.4m)}{(-9.80m/s^2)}} = 4.00s$
b. v_x = 5.0m/s t = 4.00s	$v_x = \frac{d_x}{t}$ $v_x t = d_x$ $d_x = v_x t = (5.0m/s)(4.00s) = 20m$

c. Now let's find v_{fy}. v_{iy} = 0m/s d_y = -78.4m a_y = -9.80m/s2 t = 4.00s We should use $v_f^2 = v_i^2 + 2ad$ As we can see from the right, v_{fy} = -39.2m/s. Note that we need to pick the negative root.	$v_f^2 = v_i^2 + 2ad$ $v_f^2 = (0m/s)^2 + 2ad$ $v_f^2 = 2ad$ $v_f = \pm\sqrt{2ad}$ $v_f = \pm\sqrt{2(-9.80m/s^2)(-78.4m)}$ $v_f = -39.2m/s$ $v_{fy} = -39.2m/s$

5. A ball is projected horizontally at 21 m/s from a point 40 m above the ground. Determine:
a. the horizontal distance travelled by the ball before hitting the ground.
b. the ball's instantaneous velocity as it hits the ground.
c. the angle at which the ball hits the ground

a. To find the horizontal distance we'll need to find out how long the ball is in flight. d_y = -40m v_{iy} = 0m/s a_y = -9.80m/s^2 t = ?	$d = v_i t + \frac{1}{2}at^2$ $d = 0 + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$ $\frac{2d}{a} = t^2$ $t^2 = \frac{2d}{a}$ $\sqrt{t^2} = t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-40m)}{(-9.80m/s^2)}} = 2.9s$
Now we can find the horizontal distance. v_x = 21m/s t = 2.9s	$v_x = \frac{d_x}{t}$ $v_x t = d_x$ $d_x = v_x t = (21m/s)(2.9s) = 61m$
b. Now let's find v_{fy}. d_y = -40m v_{iy} = 0m/s a_y = -9.80m/s^2 t = ? We should use $v_f^2 = v_i^2 + 2ad$ As we can see from the right, v_{fy} = -28m/s. Note that we need to pick the negative root.	$v_f^2 = v_i^2 + 2ad$ $v_f^2 = (0m/s)^2 + 2ad$ $v_f^2 = 2ad$ $v_f = \pm\sqrt{2ad}$ $v_f = \pm\sqrt{2(-9.80m/s^2)(-40m)}$ $v_f = -28m/s$ $v_{fy} = -28m/s$

The ball's instantaneous velocity as it hits the ground can be found with the Pythagorean theorem.	$v_{fx}{}^2 + v_{fy}{}^2 = v_{total}{}^2$ $v_{total}{}^2 = v_{fx}{}^2 + v_{fy}{}^2$ $\sqrt{v_{total}{}^2} = v_{total} = \sqrt{v_{fx}{}^2 + v_{fy}{}^2}$ $v_{total} = \sqrt{(21m/s)^2 + (-28m/s)^2}$ $v_{total} = 35m/s$
c. To solve part c we need to find theta. We could use sin, cos, or tan, but I'll use tan.  So the ball hits the ground at an angle of 53° below the horizontal.	$\tan\theta = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{28m/s}{21m/s}$ $\tan\theta = \frac{28m/s}{21m/s}$ $\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{28m/s}{21m/s}\right)$ $\theta = \tan^{-1}\left(\frac{28m/s}{21m/s}\right) = 53°$